Numerical study of the nonlinear refraction by an optical Kerr effect thin layer in resonant regime near the critical angle of incidence

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ABSTRACT

In this work, we have used the numerical Gaussian model to determine the profile of the local field (in three dimensions) within an optical Kerr effect thin layer and to applying this model also to study the nonlinear transmittance of a thin layer. The layer is illuminated by a Gaussian beam near the angle of total reflexion in resonant regime. Indeed, the nonlinear transmittance is very sensitive to the optical Kerr effect near the total angle of incidence.

Keywords: optical Kerr effect, nonlinear thin layer, nonlinear transmittance, all optical switching

1. Introduction

Nonlinear thin film with a positive optical Kerr effect, in grazing incidence, has been studied by several authors by the plane-wave model [1,2] or by the Gaussian model [3,4]. The majority of these studies consider the thin layer in the case of total reflection at low intensity and without multiple internal reflections (non-resonant mode). In this article, we consider a Gaussian beam in incidence slightly less than the limit angle at low-intensity in the resonant regime (multiple reflections occur within the thin layer). Principally, we study the three-dimensional profile of the local field within the thin layer and the non-linear transmittance of a thin layer of CS_2. We assume the thin layer without absorption.

This study contributes to the understanding and the applications of the nonlinear propagation within a thin layer, mainly, in the three following important domains of applications: all optical switching: [5-7], the direct determination of the Kerr coefficient (n_2) [8, 9, 10] and the microcavities which become essential for many studies and applications particularly in optoelectronics and optical communication [11, 12].

To carry out this study, we consider an optical Kerr effect thin layer with thickness e and an intensity-dependent refractive index, given by: $n = n_0 + n_2 I$, where n_0 and n_2 are respectively, the linear index and Kerr coefficient and I the local intensity. Two identical linear media whose index of refraction n_1 is slightly higher than n_0 surround the layer (figure.1). The layer is illuminated by a Gaussian laser beam, in TE mode, under an angle of incidence θ_1 slightly lower than the critical angle of incidence $\theta_c = \arcsin(n_0/n_1)$.

2. Theoretical model

The electrical field in the cavity can be separated to its two components: forward field U and the backward field V. In a nonlinear medium, the two components are coupled and the Maxwell equations

$$-2jk\frac{\partial U}{\partial y}+2j\beta\frac{\partial U}{\partial x}+\frac{\partial^2 U}{\partial x^2}=A_{NL}\left(|U|^2+2|V|^2\right)U^2 \quad (1)$$

are given by the two following nonlinear partial differential equations [13,14]:

$$2jk\frac{\partial V}{\partial y}+2j\beta\frac{\partial V}{\partial x}+\frac{\partial^2 V}{\partial x^2}=A_{NL}\left(|V|^2+2|U|^2\right)V^2 \qquad (2)$$

With,

$$\beta=\frac{\omega}{c}n_i\sin(\theta_i)=\frac{\omega}{c}n_t\sin(\theta_t),$$

$$k=\left[\left(\frac{\omega}{c}n_0\right)^2-(\beta)^2\right]^{1/2},$$

$$A_{NL}=-2\frac{\omega^2}{c^2}n_0n_2.$$

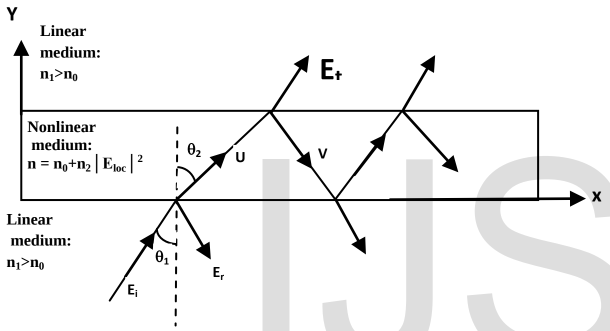

Fig.1. Nonlinear optical Kerr effect thin film ($n = n_0 + n_2 |E_{loc}|^2$). E_i: Gaussian incident field; U : forward field; V : backward field; and E_t : transmitted field and E_r : reflected beam.

Initial condition

We consider that the beam undergoes multiple reflections within the thin layer (resonant case). Because the incidence is oblique, we can consider that only the first two reflections are important.

We consider that the incident field is a Gaussian laser beam focused at the first interface (linear medium / thin film). Thus, the electric field amplitude of the incident Gaussian beam at the interface of the thin layer ($y = 0$) is given by the expression:

$$E(x,y=0)=E_i e^{-(\cos(\theta_1)x/w_0)^2}, \qquad (3)$$

Where E_i is the amplitude of the incident beam, θ_1 is the angle of incidence, and W_0 is the beam-waist at the entry of the thin film.

Boundary conditions

The two boundary conditions are defined at the interfaces of the thin layer: $y = 0$ and $y = e$, by the relations:

$$U(x,y=0)=r_1V(x,y=0)+t_1E_i(x)$$

$$V(x,y=e)=r_2U(x,y=e)$$

,
where, r_1 (r_2) and t_1 (t_2) are the coefficients of reflection and transmission of the two layer interfaces [15].

The fields U and V are assumed to be zero at the limits $x = x_{min}$ and $x = x_{max}$: of the propagation of the local field:

$$U(x=x_{\min,\max},y)=V(x=x_{\min,\max},y)=0.$$

The propagation and behavior of the local field within the thin layer and the profile of the transmitted field at the output of the layer are entirely determinate by the solution of this coupled system of nonlinear *PDE* above. We have used the second order finite difference method which is simple in writing, reliable, and very suitable for Cartesian grids.

Our Cartesian calculation domain, is defined by two variables x and y in the x-y plane.

In the x-axis: $x_{\min}\le x\le x_{\max}$, where

$$|x_{\max}-x_{\min}|=w_0/\cos(\theta_1),$$

and $\Delta x = ((x_{max} - x_{min}) / N_x)$

In the y-axis, $(0 \le y \le \boldsymbol{e})$ with $\Delta y = (\boldsymbol{e} / N_y)$, where $\boldsymbol{e}$ is the layer thickness.

N_x and N_y are the numbers of steps according to both
x-axis and y-axis.

During several numerical calculation tests, we have observed that the stability and the execution times depend mainly on the discretisation parameters: N_x, N_y, x_{min}, and x_{max}. The choice of values for these parameters was not easy because there is no systematic way to define them (mainly in nonlinear case). Thus, we start with arbitrary values and proceed to adjust these parameters one by one by turning the program several times. Every time we vary the beam intensity or the angle of incidence we should adjust these parameters. The numerical calculation stability was provided by the regularity of the obtained graphs and by the rapprochement with the experimental results.

3. Local Field

We search to describe the local field within the thin layer in function of incident intensity and the angle of incidence near the total reflexion angle θ_c for the resonant case. For this purpose, we have choose three angles of incidence (θ_i) very close to the critical angle $(\theta_i \leq \theta_c)$ and we have divided the interval of intensity in this study to subintervals in order to highlight and explain the non-linear behavior of the laser Gaussian beam during its travel within the thin layer. Because, we take into account only the first two transmitted rays, the local field is reduced to three components: U_1, V_1 and U_2 (figure .2).

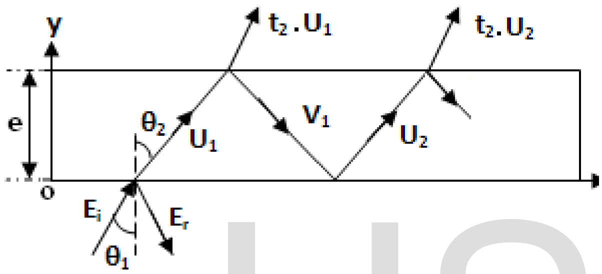

Fig.2. The local field is the sum of the first incoming field U_1, the first backward field V_1, and the second incoming field U_2.

For the numerical study, we considered a thin layer whose parameters are close to those studied experimentally [8]:
$n_0=1.608$; $n_1=1.618$; $e = 10\mu m$; $w_0 = 80\mu m$; $\lambda= 0.532\ \mu m$.
Thus, the total reflection angle is $\theta_c = 80.327°$.
The total local field at any point $M\ (x, y)$ in the thin layer is obtained from the sum of the three fields:

$$E_{loc} = U_1(x,y) + V_1(x,y) + U_2(x,y), \qquad (6)$$

where, $U_1(x,y)$ is the first forward field inside the thin layer, $V_1(x,y)$ is the backward field, and $U_2(x,y)$, is the second forward field.
From the incident beam E_i, we deduce the first local field at the first interface: $U_1\ (x, y = 0) = t_1\ E_i$,
where t_1 is the transmission coefficient of the first interface.
By iteration, we determine the first forward field inside the thin layer: $U_1\ (x, y)$ and in particular $U_1\ (x, y = e)$ from which we deduce the first backward field at the layer second interface: $V_1(x,\ y = e) = r_2 . U_1(x,\ y = e)$, where, r_2 is the reflection coefficient of the second interface.

From the backward field at the first interface of the layer, we deduce the second incoming field: $U_2(x, y = 0) = r_1 . V_1(x, y = 0)$, and by iteration we determine the entire second incoming field $U_2(x, y)$ in any point within the thin layer.

3.1. Local field versus the incident beam

Our purpose is to study the profile of the local field (propagating beam) inside the thin layer in function of the intensity of the incident beam for the incident angle $\theta_1 = 80.18°$ which is slightly inferior to the critical angle.
For the numerical calculation, we considered a nonlinear medium with an ordinary Kerr coefficient: $n_2 = 2.5\ 10^{-20}\ m^2 / W$. This is a relatively low value but is close to that of glass [16]. This study allows to know the distortions that can undergo the profile of the laser beam as it propagates inside an amplifying medium (glass).
We have carried out a series of studies by increasing the incident intensity I_i. In this study we have selected three important cases.
The study is performed as a function of the electric field E_i of the incident wave. The optical intensity I of the electrical field E_i, is giving by:

$$I_i = \frac{1}{2} n_1 c \varepsilon_0 \left|E_i\right|^2$$

The values of the electric field E_i are in Megavolt per meter (MV/m).
For an incident field amplitude $E_i = 32\ MV/m$ ($I_i = 0.193\ GW / cm^2$) which is relatively low to produce non-linear effects, the three dimension figure (3) shows an overall decrease and increase in the local field. The cross-sections show in greater detail the field profile along its propagation in the layer. It is observed that the Gaussian profile of the field is constant; the amplitude decreases to $20\ MV/m$ and then goes back to its initial value $32\ MV/m$. In some figures (3.b), we observe the appearance of negative values of the field. The appearance of these negative values can be understood by the fact that the backward field is negative. In fact, the reflection coefficient r_2 of the second interface is negative. The figure (4) corresponds to the threshold amplitude $E_{is} = 50\ MV/m$ ($I_{is} = 0.6736\ GV/ cm^2$) at which the nonlinear effects appear: The beginning of an asymmetric fragmentation (on a side) of the field profile is observed at the output of the thin film. The corresponding cross-sectional figures show that the field impulsions present some changes in amplitudes and profiles throughout the thin layer. The figure (5) shows the local field propagation with important non-linear effects. The corresponding field amplitude is equal to $E_i = 62\ MV/m$ ($I_i = 0.8257\ GW/cm^2$). The cross-sections

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show the fragmentation of the Gaussian profile of the field into two pulses *(at 6 mμ)* and three pulses at the output of the thin layer. This effect was theoretically and experimentally observed [3, 8].

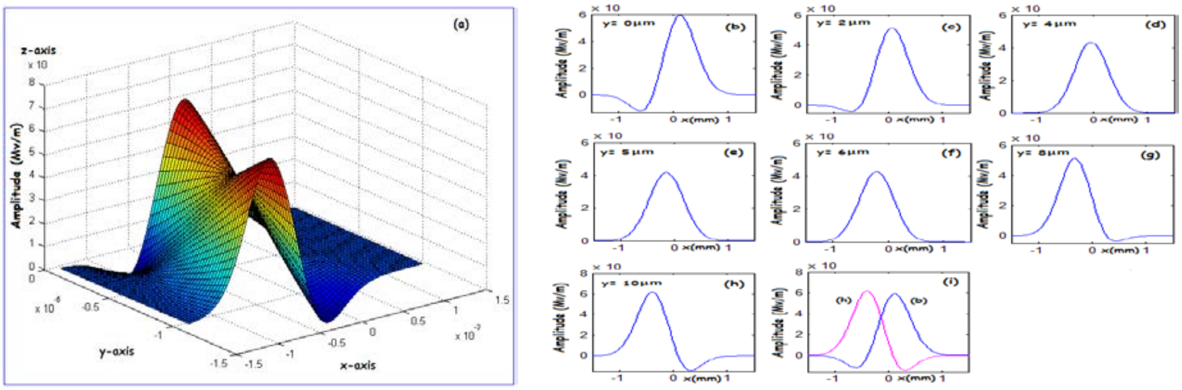

Fig.3. Profile of the local electric field inside the thin layer corresponding to the incident field E_i = 32 MV / m (E_i <E_s). a: 3-D representation; b-i: cross sections.

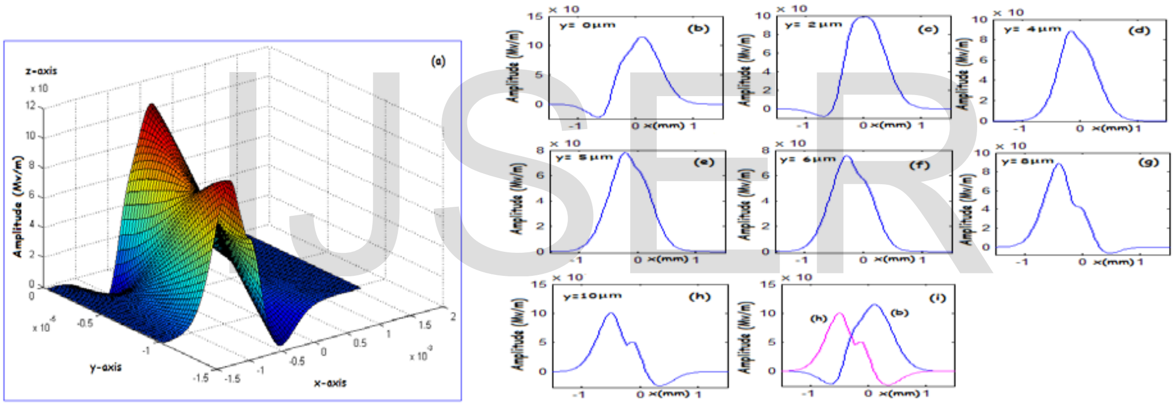

Fig. 4. Profile of the local electric field inside the thin layer corresponding to the incident field E_i = E_{is} = 50 MV/m. a: 3-D representation; b-i: cross sections.

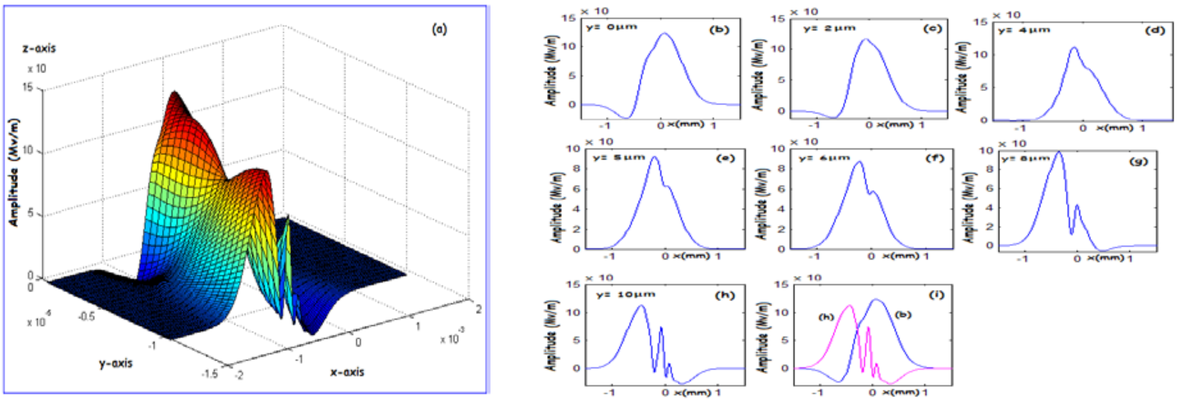

Fig. 5. Profile of the local electric field inside the thin layer corresponding to the incident field E_i = 62 MV/m ($E_i > E_{is}$). a: 3-D representation; b-i : cross sections.

3. Nonlinear transmittance

The study of the nonlinear transmittance (T_{NL}) of the thin layer in function of the incident intensity I_i ($I_i \propto \left|E_i^2\right|$) and the angle of incidence *(θ_i)* is very important for the use of this thin layer in nonlinear applications. The nonlinear transmittance (T_{NL}) is the potential basic element for the realization of all optical components [17,18].

For this raison, we present the numerical study of the optical nonlinear transmittance T_{NL} *(transmitted intensity / incident intensity)* of the thin film by the Gaussian model in the resonant case. However, the nonlinear transmitted field has a distorted spatial profile and hence an implicit expression. Thus, we have to calculate by integration all the energy contained in the envelope of the beam profile:

$$T = \frac{\int_{x_{\min}}^{x_{\max}} \left|E_t(x)\right|^2 dx}{\int_{x_{\min}}^{x_{\max}} \left|E_i(x)\right|^2 dx} \quad (7)$$

Where, and $E_t(x)$ and $E_i(x)$ are the transmitted and the incident fields, respectively and x_{min} and x_{max} are the boundaries of the beam.

At the layer output, the transmitted field is related to the local field by:

$$E_t(x) = t_2 U_1(x, y = e) + t_2 U_2(x, y = e)$$

Where, U_1 and U_2 are the first and the second entering fields in the thin layer, respectively and t_1 and t_2 are the transmission coefficient of both first and second interfaces of the layer and *e* is the layer thickness.

CS_2 is a standard reference material in nonlinear optics for determining the Kerr coefficient (n_2), the non-linear susceptibility of the third order and in other experimental studies.

To test the effectiveness and stability of the performed numerical calculations, we made a comparison with experimental results previously made. [8]. The CS_2 thin layer of is identical to that discussed above.

The experimental study [8] of the non-linear transmittance $(T_{NL})_{exp}$ of CS_2 in function of the incident intensity I_0 is shown in Figure (7.a).

For the numerical computation, we have considered the CS_2 Kerr coefficient $n_2 = 2.5.\ 10^{-18}\ m^2/W$. Figure (7.b) shows the variations of the $(T_{NL})_{num}$ in function of the incident intensity I_0.

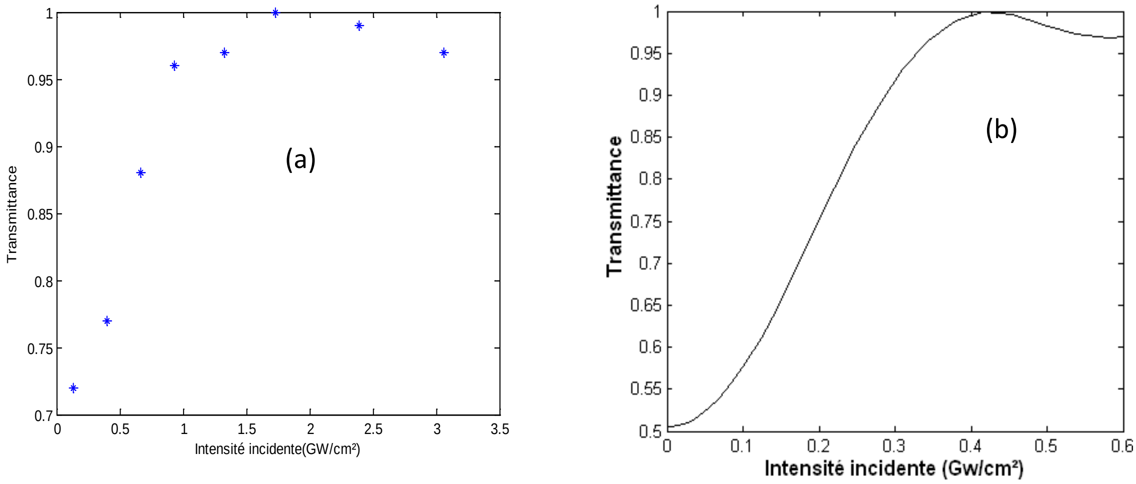

Fig.7.. Nonlinear transmittance of a CS_2 thin layer. (a): experimental [8], (b): by numerical Gaussian model.

The two transmittances in figure (7) have identical behavior but their variation intervals of the incident intensity are different. The incident intensities are larger in the case of the experimental nonlinear transmittance, which implies a lower sensitivity (S_{exp}). This effect can be explained by the hypothesis that the incident beam in the experimental case is not perfectly Gaussian, which is often the case in practice. The transmittance of an optical Kerr effect thin layer becomes very sensitive to variations of the refractive index near the angle of total reflection. Any increase in the incident intensity causes a change in the index ($n = n_0 + n_2 I$) of the layer and hence of the system. For a certain value of the incident intensity, the transmittance of the layer changes from *low state: (0)* to the *high state: (1).*
With such an operation a thin layer may be used to perform all-optical switching or all optical limitation [19]. It is the light that causes the index variations; it is then an optically controlled optical gate. The switching time depends on the optical nonlinearity an involved intrinsic response time of the cavity. By varying the thickness of the cavity, the light pulse duration, and the nature of the excitation, switching time below (*ps*) can be obtained [20]. If the low (or high) level of transmission depends on the history of anterior illumination, the system can serve as a basis for the realization of an optical memory.

5. Conclusion

By using the Gaussian model, we have studied the propagation and the behavior of the local field within a nonlinear thin layer and the transmittance of the CS_2 thin layer. This study shows some important results:
The non linear effect affects the local field propagation when the incident intensity is greater than a threshold value. The field fragmentation is asymmetric and begins on a side of the field profile. If we increase the intensity even more, the field profile splits into two pulses or more. As the angle of incidence approaches the critical one the nonlinear effect becomes more important and its threshold intensity decreases and the difference in amplitude between the incident and the local fields, decreases, which implies an increase of the transmittance.
We used numerical study to determine the transmittance of a thin layer of CS_2 which was previously studied experimentally by Mountasser et al [8]. The results show that the two transmittances are closely similar. This makes it possible to confirm the validity of the numerical model.

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